SM3 10.4 Infinite Geometric Series

If the series converges, find the sum of the geometric series. Otherwise, state that it diverges.

1) $1 + 2 + 4 + 8 + \cdots$

The series diverges.

2) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

The series converges to 1.

3) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$

The series converges to $\frac{3}{4}$.

4) $2 + \frac{4}{5} + \frac{8}{25} + \frac{16}{125} + \cdots$

The series converges to $\frac{10}{3}$.

5) $\sum_{n=1}^{\infty} 3\left(\frac{2}{5}\right)^{n-1}$

The series converges to 5.

6) $\sum_{n=1}^{\infty} 7\left(\frac{4}{3}\right)^{n-1}$

The series diverges.

7) $\sum_{n=1}^{\infty} 2\left(-\frac{1}{3}\right)^{n-1}$

The series converges to $\frac{3}{2}$.

8) $\sum_{n=1}^{\infty} \frac{1}{7}\left(\frac{4}{9}\right)^{n-1}$

The series converges to $\frac{9}{35}$.

Write the series using sigma (Σ) notation. Then, if the series converges, find the sum of the geometric series. Otherwise, state that it diverges.

9) $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots$

$$\sum_{n=1}^{\infty} 1\left(\frac{2}{3}\right)^{n-1} = 3$$

10) $5 - 25 + 125 - 625 + \cdots$

$$\sum_{n=1}^{\infty} 5\,(-5)^{n-1}$$

which is a divergent series

11) $\frac{25}{64} + \frac{5}{16} + \frac{1}{4} + \frac{1}{5} + \ldots$

$$\sum_{n=1}^{\infty} \frac{25}{64}\left(\frac{4}{5}\right)^{n-1} = \frac{125}{64}$$

12) $\ldots + \frac{1}{25} + \frac{1}{5} + 1 + 5$

$$\sum_{n=1}^{\infty} 5\left(\frac{1}{5}\right)^{n-1} = \frac{25}{4}$$

Write the repeating decimal as a simplified fraction:

13) $0.\overline{5}$

$\frac{5}{9}$

14) $0.\overline{37}$

$\frac{37}{99}$

15) $0.\overline{111}$

$\frac{1}{9}$

16) $0.\overline{6472}$

$\frac{6472}{9999}$

17) $1.\overline{58}$

$1+\frac{58}{99}$

$\frac{157}{99}$

18) $2.\overline{438}$

$2+\frac{438}{999}$

$\frac{2436}{999}=\frac{812}{333}$

19) As a brand new driver, Cassie runs into 20 parked cars in her neighborhood at age 16. She only runs into 10 parked cars in her neighborhood while she is 17, perhaps because her skill increases or perhaps because her neighbors are too terrified to leave their cars parked on the street! The pattern continues each year and the number of parked cars she runs into each year is only half of the number of parked cars during the previous year. If she were to live forever, smashing into parked cars until the end of time, with how many parked cars would Cassie have collided?

$$c_1 = 20, r = \frac{1}{2}$$

$$s = \frac{c_1}{1-r} = \frac{20}{1-\frac{1}{2}} = \frac{20}{\frac{1}{2}} = 40$$

20) You drop a particularly bouncy ball from a height of 80 feet. The ball is so bouncy, that each time it hits the ground, it returns to a height that is $\frac{3}{4}$ of the most recent height. As the ball is continues bouncing, what is the total distance travelled by the ball approaching? (*hint: the ball moves both up and down, which makes the problem more complicated*)

$$2S - a_1 = 2\left(\frac{a_1}{1-r}\right) - a_1 = 2\left(\frac{80}{1-\frac{3}{4}}\right) - 80 = 2\left(\frac{80}{\frac{1}{4}}\right) - 80 = 2(320) - 80 = 640 - 80 = 560$$